Graphing a Straight Line Using Slope Intercept Technique

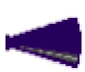
The discussion below walks you through Problem 1 with these steps:

- **Step 1: Solve for y.**
- **Step 2: Locate the y intercept on a graph.**
- **Step 3: Starting at the y intercept, use the slope to draw a right triangle that allows you to determine a second point on the line.**
- **Step 4: Draw the straight line through the two points.**
- **Step 5: Interpret the slope and y intercept of a linear equation.**

The discussion uses several formats:

- visual (with pictures)
- symbolic (with equations)
- verbal (with text)
- aural (with audio)

When you see the megaphone icon (below), **Click / Select** it to listen to an audio file that provides additional information.



Audio

Graphing a Straight Line Using Slope Intercept Technique

The equation below uses the statistical **Slope Intercept form**:

$$y = bx + a$$

If you have recently studied algebra, you may remember this form:

$$y = mx + b$$

For the discussion below, we will use the statistical form. Both forms mean the same thing even though they use different letters.

Note that b and a are constants that can be positive, negative, whole, fraction, or decimal numbers. For the problem, they do not change in value once they have been determined.

Each straight line has one slope and one y intercept.

$$Slope = b = \frac{\Delta y}{\Delta x} = \frac{Change\ in\ y}{Change\ in\ x}\ or\ \frac{Delta\ y}{Delta\ x}$$

The slope explains how the variables x and y are **related.**

Systems thinking says: Everything is related to everything else.

Audio

The slope gives us a **quantitative** explanation of this relationship.

y intercept = a $\Rightarrow$ (0, a)

The y intercept is the point on the graph where the line crosses the y axis.
It is the y value when x = 0.

Problem One: Graph 6x + 2y = 10

Step 1: Solve for y.

2 y = - 6x + 10

$$\frac{2y}{2} = \frac{-6x}{2} + \frac{10}{2}$$

y = -3x + 5

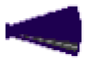
In this form, we can easily determine the slope and y intercept of the line.

y = b x + a
y = -3x + 5

$$Slope = b = \frac{\Delta y}{\Delta x} = -3 = \frac{-3}{1} = \frac{3}{-1}$$

y intercept = a = 5 ⇒ (0, 5)

Step 2: Locate the **y intercept** on a graph.



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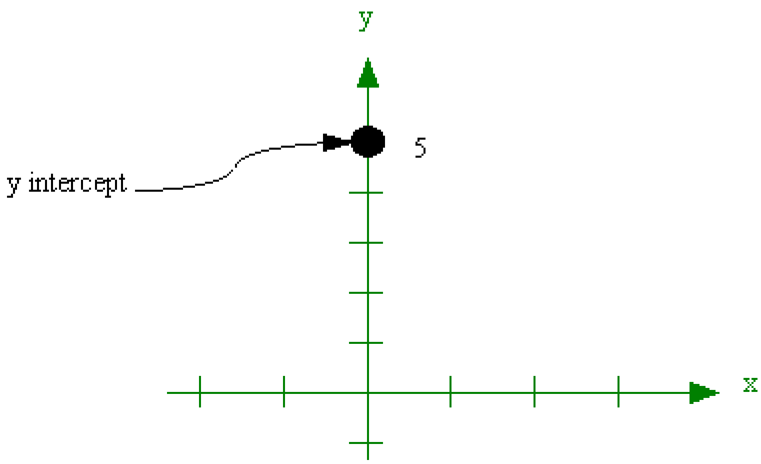

Step 3: Starting at the y intercept, use the slope to draw a right triangle that allows you to determine a second point on the line.

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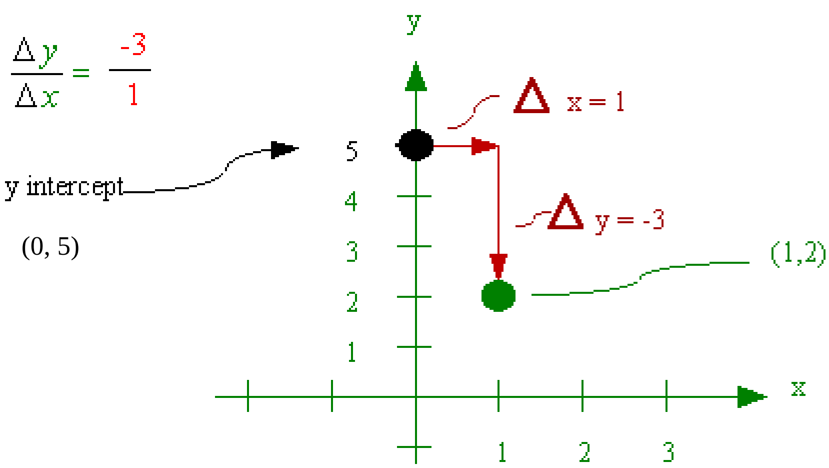
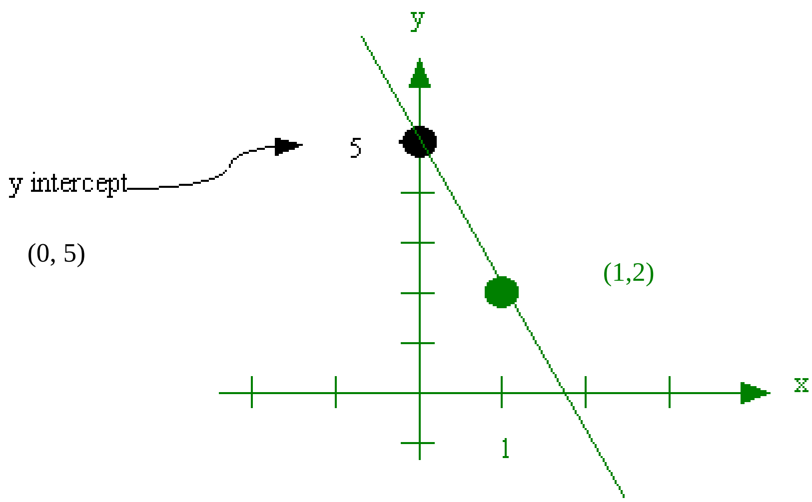


Step 4: Draw the straight line through the two points.



The line shows all solution points (x, y) for the equation 6x + 2y = 10.

Since a line is made up of an infinite number of points, there are an infinite number of solutions to the equation:

$$6x + 2y = 10$$

Note, since the slope (-3) also equals

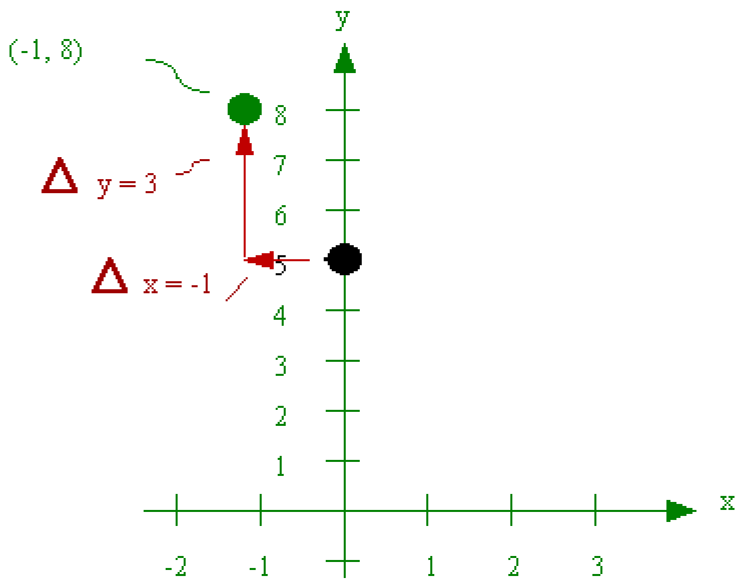
$$\frac{3}{-1}$$

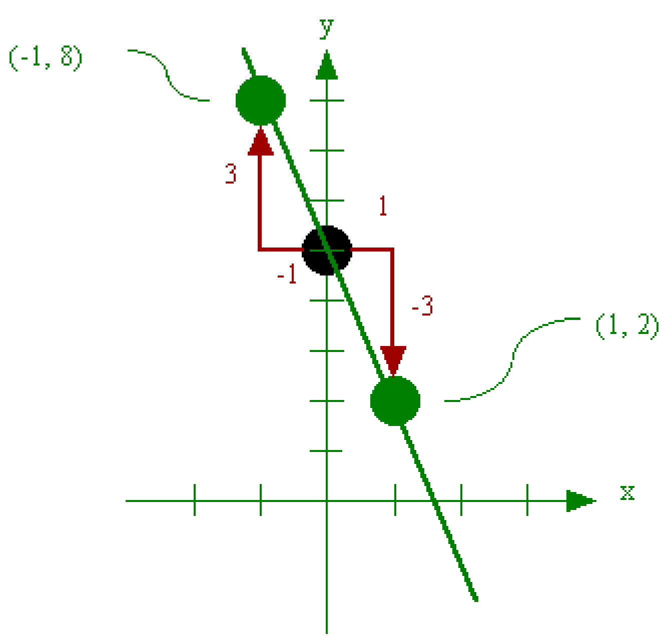
you could have started at the y intercept and drawn a different right triangle. Two representations of the slope (that mean the same thing) are used to draw the graphs below.

Audio

As noted above, these two graphs both show that the slope is −3.

$$\frac{3}{-1} = \frac{-3}{1} = -3$$





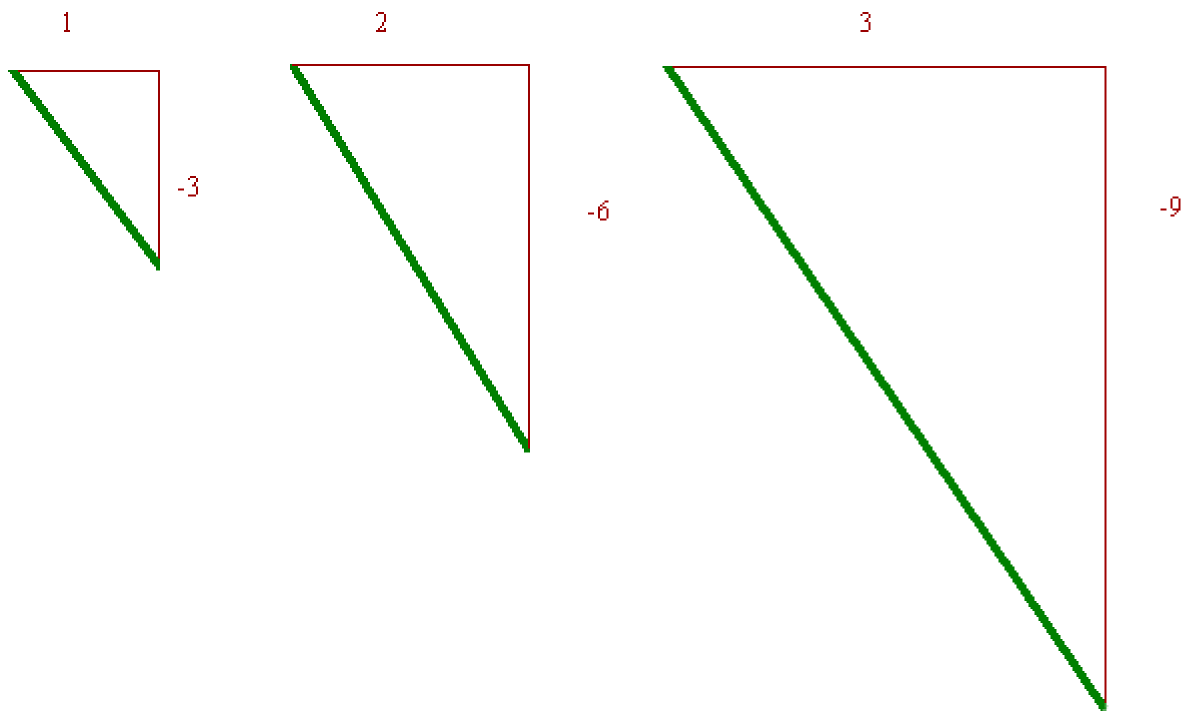
Remember: There is only one slope to a straight line.

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \text{ is a constant}$$

Since $\frac{\Delta y}{\Delta x}$ is a fraction, it can be written in an infinite number of ways:

$$-3 = \frac{-3}{1} = \frac{-6}{2} = \frac{-9}{3} = \ldots$$

These right triangles are similar triangles: they all will give the same straight line.



Step 5: Interpret the slope and y intercept of a linear equation.

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Assume Regression Analysis has given the following results:

x = $ spent on advertising (100)
y = Sales $ (1000)

Ŷ = .75x + 12,   0 ≤ x ≤ 10

The addendum 0 ≤ x ≤ 10 tells us the model is based on advertising levels that have gone from $0 to $10,000 (10 x 1000). Regression demonstrates Kant's view of where some knowledge comes from. It is a wonderful example of how, when reasoning is blended with our experiences correctly, one is able to create knowledge about how variables are related. Regression is a summary of our experiences; it may be inappropriate to use the model to predict sales for advertising levels outside this level of experience.

The "hat" (^) on the y stands for "the estimate of."

Ŷ reads, "Our estimated sales in $1,000 units."

Interpretation of the slope:

$$slope = \frac{\Delta y}{\Delta x} = .75 = \frac{.75}{1} \text{ or } \frac{75}{100} \text{ or } \frac{3}{4} \text{ etc.}$$

$$slope = \frac{\text{Change in Sales } \$(1000)}{\text{Change in Ad. } \$(100)} = \frac{.75}{1}$$

Oftentimes, to make the numbers easier to work with, zeros are dropped in the calculations. Then to correctly interpret the results, one must put the zeros back in.

Now we must adjust for the zeros that have been dropped. x is in $100 units and y is in $1000 units. Multiply (1 x 100) for Advertising; and (.75 x 1000) for Sales to get the following:

$$slope = \frac{\$750 \; in \; Sales}{\$100 \; in \; Advertising}$$

Interpretation of Slope: It has been our experience for advertising levels from $0 to $10,000, that with each additional $100 spent on advertising, we have experienced a $750 increase in sales. For yet another interpretation, listen to the Audio file below.

Audio

Interpretation of y intercept: The y intercept, "a" is the y value when x = 0. For this problem, the y intercept is the sales $ we'd expect when we do no advertising.

When x = 0: $\hat{Y} = .75(0) + 12 = 12$

Adjusting the zeros: $\hat{Y} = 12 \times 1000 = 12{,}000$

Since we have experiences with no advertising,

$$0 \leq x \leq 10$$

x = 0

we can say: It has been our experience that when there is no advertising, sales have been about $12,000.

Note: If the addendum had been different,

$\hat{Y} = .75x + 10, \quad 3 \le x \le 10$

the interpretation of the y intercept would be: Since we have no experience with no advertising (x = 0), we will not speculate on what would happen with no advertising.

Often people speculate on measuring the y intercept even though they have no experience. This can be very “dangerous.” Always look to see if there are assumptions—and then evaluate them.

Practice Problem: Graph $4x - 3y = 9$

We suggest you try this problem on your own and then check your answer with ours (below).

Practice Problem Answer